Theoretical Considerations Regarding the Application of Received Signal Strength within Heterogeneous Indoor Positioning Systems

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***Abstract*—Nowadays, there are a variety of different indoor positioning systems, where some of them use communication hardware taking advantage of the Received Signal Strength (RSS) such as Wireless Local Area Networks (WLAN) or Bluetooth. These variants are employed if low cost is of primary importance. However, the accuracy provided is in the meter range. The alternative are positioning-tailored approaches like Frequency Modulated Continuous Wave (FMCW) radar, Ultra-WideBand (UWB) radar or phase-based positioning, which offer superior accuracy in the low decimetre range. If there is such a system in use, the question arises whether there is any improvement, if utilizing additional RSS measurements, which are performed by most systems anyway. With the help of the Cramér-Rao Lower Bound (CRLB), this paper demonstrates that these additional readings can improve accuracy significantly, thus widen the application field for RSS from a low-budget only technique to enabling enhanced accurate positioning. To demonstrate this statement we compare the CRLB for Time of Arrival (ToA) with hybrid ToA/RSS. Our evaluations show that in practice the CRLB is approximately divided by two, if incorporating the RSS for each base station.**

Keywords*—*Localization, Positioning, Cramér Rao Lower Bound, CRLB, Time of Arrival, ToA, Received Signal Strength, RSS, Hybrid ToA/RSS

I. Introduction

TODAY there are two main fields of research for indoor positioning systems. On the one hand, there are techniques taking advantage of already available infrastructure to estimate the unknown position of a user. These approaches frequently utilize the RSS of systems originally set up for communication, e.g. WLAN [1], Bluetooth [2] or ZigBee [3] with mean positioning error above 1 m. Although we limit our considerations within this paper to RSS, there are other related variants. As an example, a system employing the Internet Protocol (IP) addresses for geo-localization is presented in [4]. In short, in this category the underlying hardware is not altered but reused.

The alternative are approaches particularly built for positioning. Examples include FMCW radar [5], UWB radar [6] or ZigBee phase-based positioning [7]. At the expense of sophisticated infrastructure, they offer superior results with positioning errors in the low decimetre range.

Within this paper we show that even though RSS-based variants are inferior, their incorporation can significantly increase the accuracy for position-tailored techniques. These RSS measurements are usually generated anyway, e.g. to check if a minimal receiving power is available to perform ranging measurements.

The rest of this paper is organized as follows. Section II presents the mathematical basics. The CRLB for selected variants are derived in section III and IV. In the next section V, the CRLBs are compared by means of evaluations and the initial claim is verified. The last section VI concludes the paper.

II. Statistical Foundations

In the further course of this paper we use the following designators. A vector is denoted by a bold lower italic letter (e.g. $\boldsymbol{b}$), whereas matrices employ bold italic capital letters (e.g. $\boldsymbol{B}$). The symbol $\tilde{\cdot}$ indicates random variables (e.g. $\tilde{X}$) and the prefix ${}^{E}\cdot$ is used to characterize an estimator, which is generally also a random variable (e.g. ${}^{E}\tilde{X}$).

The CRLB specifies a lower bound for the covariance matrix of any unbiased estimator ${}^{E}\tilde{\boldsymbol{\theta}}$ for the unknown parameter $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{\mathrm{N}})^T$, i.e. [8]

$$\mathbb{COV}\left[{}^{E}\tilde{\boldsymbol{\theta}}\right] - \boldsymbol{F}^{-1}(\boldsymbol{\theta}) \geq 0 \tag{1}$$

Here, $\mathbb{COV}\left[{}^{E}\tilde{\boldsymbol{\theta}}\right]$ denotes this covariance matrix. Moreover, the inverse of the Fisher information matrix $\boldsymbol{F}(\boldsymbol{\theta})$ is required, where the element in the i-th row and j-th column reads

$$[\boldsymbol{F}(\boldsymbol{\theta})]_{(i,j)} = -\mathbb{E}\left[\frac{\partial^2 \ln\left(f(\tilde{\boldsymbol{M}}|\boldsymbol{\theta})\right)}{\partial\theta_i \partial\theta_j}\right] \tag{2}$$

Above, we have taken into account that the unknown $\boldsymbol{\theta}$ might only be determined by means of some intermediate measurement vector $\tilde{\boldsymbol{M}} = \left(\tilde{M}_1, \ldots, \tilde{M}_{\mathrm{N}}\right)^T$.

III. Cramér Rao Lower Bounds

Below, we derive the CRLB for ToA, RSS and ToA/RSS, since the CRLB specifies the theoretical optimum for any unbiased estimator utilizing these approaches. This enables to evaluate the performance for an arbitrary positioning system utilizing one of these techniques.

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A. Time of Arrival

The CRLB is derived for the localization of a Mobile Station (MS) which measures the transmission time $t_{i\leftrightarrow\bullet}$ to every Base Station (BS) i, where $i = 1, \ldots, \mathrm{N}$. We assume that these readings are Gaussian with constant measurement variance σ_T^2 for all BS, i.e.

$$\tilde{t}_{i\leftrightarrow\bullet} \sim \mathcal{N}\left(\frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}, \sigma_\mathrm{T}^2\right) \tag{3}$$

The associated true distance to the i-th BS is denoted as $d_{i\leftrightarrow\bullet} = \sqrt{(x_i - x_\bullet)^2 + (y_i - y_\bullet)^2}$, where $(x_\bullet, y_\bullet)$ is the sought unknown true position of the MS. In eq. (3), c is the speed of light. Hence, the conditional probability density function (p.d.f.) can be written as

$$f(\tilde{t}_{i\leftrightarrow\bullet}|x_\bullet, y_\bullet) = \frac{1}{\sqrt{2\pi}\sigma_\mathrm{T}} \exp\left(-\frac{1}{2\sigma_\mathrm{T}^2}\left[\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right]^2\right) \tag{4}$$

It is reasonable to assume that all N single measurements $\tilde{t}_{i\leftrightarrow\bullet}$ are stochastically independent. Thus, the joint p.d.f. reads

$$f(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet) = \prod_{i=1}^{\mathrm{N}} f(\tilde{t}_{i\leftrightarrow\bullet}|x_\bullet, y_\bullet) \tag{5}$$

where $\tilde{\boldsymbol{t}} = (\tilde{t}_{1\leftrightarrow\bullet}, \ldots, \tilde{t}_{\mathrm{N}\leftrightarrow\bullet})^T$ is a vector combining these time measurements. For determining the CRLB, we calculate the logarithmic joint p.d.f. $l(\tilde{\boldsymbol{t}}|x\bullet, y\bullet)$, which is defined as

$$\begin{aligned} l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet) &:= \ln\left(f(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)\right) \\ &= \ln\left(\prod_{i=1}^{\mathrm{N}}\left\{\frac{1}{\sqrt{2\pi}\sigma_\mathrm{T}}\exp\left(-\frac{1}{2\sigma_\mathrm{T}^2}\left[\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right]^2\right)\right\}\right) \\ &= \mathrm{N}\cdot\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\mathrm{T}}\right) - \frac{1}{2\sigma_\mathrm{T}^2}\sum_{i=1}^{\mathrm{N}}\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)^2 \end{aligned} \tag{6}$$

Above, eq. (4) and (5) are employed along with the well-known logarithmic identities. According to eq. (1), the inverse of

$$\boldsymbol{F}(\boldsymbol{\theta}) = \boldsymbol{F}(x_\bullet, y_\bullet) = \begin{bmatrix} F_{x_\bullet x_\bullet} & F_{x_\bullet y_\bullet} \\ F_{y_\bullet x_\bullet} & F_{y_\bullet y_\bullet} \end{bmatrix} \tag{7}$$

is required, where the matrix elements are characterized as

$$\begin{aligned} F_{x_\bullet x_\bullet} &= -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial x_\bullet^2}\right] & F_{x_\bullet y_\bullet} &= -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial x_\bullet \partial y_\bullet}\right] \\ F_{y_\bullet y_\bullet} &= -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial y_\bullet^2}\right] & F_{y_\bullet x_\bullet} &= F_{x_\bullet y_\bullet} \end{aligned} \tag{8}$$

Performing the second order derivation for $x_\bullet$ yields

$$\begin{aligned} &\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial x_\bullet^2} \\ &= -\frac{1}{\mathrm{c}\cdot\sigma_\mathrm{T}^2}\sum_{i=1}^{\mathrm{N}}\left[\frac{(x_i - x_\bullet)^2}{\mathrm{c}\cdot d_{i\leftrightarrow\bullet}^2} + \frac{(x_i - x_\bullet)^2\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)}{d_{i\leftrightarrow\bullet}^3} - \frac{\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)}{d_{i\leftrightarrow\bullet}}\right] \end{aligned} \tag{9}$$

Equally,

$$\begin{aligned} &\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial y^2} \\ &= -\frac{1}{\mathrm{c}\cdot\sigma_\mathrm{T}^2}\sum_{i=1}^{\mathrm{N}}\left[\frac{(y_i - y_\bullet)^2}{\mathrm{c}\cdot d_{i\leftrightarrow\bullet}^2} + \frac{(y_i - y_\bullet)^2\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)}{d_{i\leftrightarrow\bullet}^3} - \frac{\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)}{d_{i\leftrightarrow\bullet}}\right] \end{aligned} \tag{10}$$

And for the mixed second order derivations for $x_\bullet$ and $y_\bullet$:

$$\begin{aligned} &\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial y_\bullet \partial x_\bullet} \\ &= -\frac{1}{\mathrm{c}\cdot\sigma_\mathrm{T}^2}\sum_{i=1}^{\mathrm{N}}\left[-\frac{(x_i - x_\bullet)\cdot(y_i - y_\bullet)}{\mathrm{c}\cdot d_{i\leftrightarrow\bullet}^2} - \frac{(x_i - x_\bullet)\cdot(y_i - y_\bullet)\left(\tilde{t}_{i\leftrightarrow\bullet} - \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}}\right)}{d_{i\leftrightarrow\bullet}^3}\right] \end{aligned} \tag{11}$$

Due to the underlying Gaussian distribution, we have

$$\mathbb{E}\left[\tilde{t}_{i\leftrightarrow\bullet}\right] = \frac{d_{i\leftrightarrow\bullet}}{\mathrm{c}} \tag{12}$$

and thus the elements of the Fisher information matrix $\boldsymbol{F}(x_\bullet, y_\bullet)$ read

$$F_{x_\bullet x_\bullet} = -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial x_\bullet^2}\right] = \mathrm{K_S}\sum_{i=1}^{\mathrm{N}}\left[\frac{(x_i - x_\bullet)^2}{d_{i\leftrightarrow\bullet}^2}\right] \tag{13}$$

$$F_{x_\bullet y_\bullet} = -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial y_\bullet \partial x_\bullet}\right] = -\mathrm{K_S}\sum_{i=1}^{\mathrm{N}}\left[\frac{(x_i - x_\bullet)(y_i - y_\bullet)}{d_{i\leftrightarrow\bullet}^2}\right] \tag{14}$$

$$F_{y_\bullet y_\bullet} = -\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_\bullet, y_\bullet)}{\partial y_\bullet^2}\right] = \mathrm{K_S}\sum_{i=1}^{\mathrm{N}}\left[\frac{(y_i - y_\bullet)^2}{d_{i\leftrightarrow\bullet}^2}\right] \tag{15}$$

To simplify, we have set above

$$\mathrm{K_S} := \frac{1}{\mathrm{c}^2\cdot\sigma_\mathrm{T}^2} \tag{16}$$

For the CRLB we need to determine the inverse of this matrix, which can be written as [9]

$$\boldsymbol{F}^{-1}(x_\bullet, y_\bullet) = \frac{1}{F_{x_\bullet x_\bullet}\cdot F_{y_\bullet y_\bullet} - F_{x_\bullet y_\bullet}^2}\begin{bmatrix} F_{y_\bullet y_\bullet} & -F_{x_\bullet y_\bullet} \\ -F_{y_\bullet x_\bullet} & F_{x_\bullet x_\bullet} \end{bmatrix} \tag{17}$$

The CRLB for an unbiased estimator $({}^E\tilde{x}_\bullet, {}^E\tilde{y}_\bullet)$ is given according to eq. (1) as

$$\mathbb{COV}\left[{}^E\tilde{x}_\bullet, {}^E\tilde{y}_\bullet\right] - \boldsymbol{F}^{-1}(x_\bullet, y_\bullet) \geq 0 \tag{18}$$

where the covariance matrix $\mathbb{COV}\left[{}^E\tilde{x}_\bullet, {}^E\tilde{y}_\bullet\right]$ reads

$$\begin{aligned} &\mathbb{COV}\left[{}^E\tilde{x}_\bullet, {}^E\tilde{y}_\bullet\right] \\ &= \begin{bmatrix} \mathbb{E}\left[({}^E\tilde{x}_\bullet - x_\bullet)^2\right] & \mathbb{E}\left[({}^E\tilde{x}_\bullet - x_\bullet)({}^E\tilde{y}_\bullet - y_\bullet)\right] \\ \mathbb{E}\left[({}^E\tilde{y}_\bullet - y_\bullet)({}^E\tilde{x}_\bullet - x_\bullet)\right] & \mathbb{E}\left[({}^E\tilde{y}_\bullet - y_\bullet)^2\right] \end{bmatrix} \end{aligned} \tag{19}$$

We seek a lower bound for the expectation of the positioning error $\mathbb{E}\left[({}^E\tilde{x}_\bullet - x_\bullet)^2 + ({}^E\tilde{y}_\bullet - y_\bullet)^2\right]$. Due to the linearity of the expectation operator, we can reduce this problem to finding

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the errors $\mathbb{E}\left[({}^{E}\tilde{x}_{\bullet}-x_{\bullet})^2\right]$ and $\mathbb{E}\left[({}^{E}\tilde{y}_{\bullet}-y_{\bullet})^2\right]$. For these, we insert inequality (18), respectively, to yield the CRLB [10]

$$\begin{aligned}
&\mathbb{E}\left[({}^{E}\tilde{x}_{\bullet}-x_{\bullet})^2+({}^{E}\tilde{y}_{\bullet}-y_{\bullet})^2\right]\\
&=\mathbb{E}\left[({}^{E}\tilde{x}_{\bullet}-x_{\bullet})^2\right]+\mathbb{E}\left[({}^{E}\tilde{y}_{\bullet}-y_{\bullet})^2\right]\\
&\geq\frac{F_{x_{\bullet}x_{\bullet}}}{F_{x_{\bullet}x_{\bullet}}\cdot F_{y_{\bullet}y_{\bullet}}-F_{x_{\bullet}y_{\bullet}}^2}+\frac{F_{y_{\bullet}y_{\bullet}}}{F_{x_{\bullet}x_{\bullet}}\cdot F_{y_{\bullet}y_{\bullet}}-F_{x_{\bullet}y_{\bullet}}^2}\\
&=\frac{\sum\limits_{i=1}^{\mathrm{N}}\left[\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]+\sum\limits_{i=1}^{\mathrm{N}}\left[\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]}{\mathrm{K_S}\left\{\begin{array}{l}\left[\sum\limits_{i=1}^{\mathrm{N}}\left[\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]\cdot\sum\limits_{i=1}^{\mathrm{N}}\left[\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]\right.\\ \left.\qquad-\left(\sum\limits_{i=1}^{\mathrm{N}}\left[\frac{(x_i-x_{\bullet})\cdot(y_i-y_{\bullet})}{d_{i\leftrightarrow\bullet}^2}\right]\right)^2\right]\end{array}\right\}}
\end{aligned}\tag{20}$$

B. Received Signal Strength

Techniques utilizing the RSS, frequently estimate the distance with the help of the Log-normal channel model

$$\tilde{P}:=\tilde{P}_{RX}/dBm=A-10\cdot\eta\cdot\log_{10}\left(\frac{d}{d_0}\right)+\tilde{N}\ ;\ \tilde{N}\sim\mathcal{N}\left(0,\sigma_{\mathrm{R}}^2\right)\tag{21}$$

Below, we derive the CRLB for these approaches. Due to the underlying Log-normal distribution, the received signal strength $\tilde{P}_{i\leftrightarrow\bullet}$ on the MS caused by a signal from BS i complies to a Gaussian distribution

$$\tilde{P}_{i\leftrightarrow\bullet}\sim\mathcal{N}\left(A_i-10\cdot\eta_i\cdot\log_{10}\left(\frac{d_{i\leftrightarrow\bullet}}{d_0}\right),\sigma_{\mathrm{R,i}}^2\right)\tag{22}$$

where $d_{i\leftrightarrow\bullet}$ is defined as before. Without loss of generality, we assume that A_i, η_i und $\sigma_{\mathrm{R,i}}$ are identical for all BS, thus we omit index i. Hence, the conditional p.d.f. reads

$$\begin{aligned}
&f(\tilde{P}_{i\leftrightarrow\bullet}|x_{\bullet},y_{\bullet})\\
&=\frac{1}{\sqrt{2\pi}\sigma_{\mathrm{R}}}\exp\left(\frac{-\left[\tilde{P}_{i\leftrightarrow\bullet}-\left\{A-10\eta\cdot\log_{10}\left(\frac{d_{i\leftrightarrow\bullet}}{d_0}\right)\right\}\right]^2}{2\sigma_{\mathrm{R}}^2}\right)
\end{aligned}\tag{23}$$

Assuming stochastically independent measurements to the N BS, we can write the joint p.d.f. with the help of the vector $\tilde{\boldsymbol{P}}=(\tilde{P}_{1\leftrightarrow\bullet},\ldots,\tilde{P}_{N\leftrightarrow\bullet})^T$:

$$f(\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})=\prod_{i=1}^{\mathrm{N}}f(\tilde{P}_{i\leftrightarrow\bullet}|x_{\bullet},y_{\bullet})\tag{24}$$

Hence, the logarithmic joint p.d.f $l(\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})$ becomes

$$l(\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet}):=\ln\Big(f(\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})\Big)\tag{25}$$

A similar derivation as in the case of ToA can be performed to obtain [11]:

$$F_{x_{\bullet}x_{\bullet}}=-\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_{\bullet},y_{\bullet})}{\partial x_{\bullet}^2}\right]=\mathrm{K_R}\sum_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\tag{26}$$

$$F_{y_{\bullet}y_{\bullet}}=-\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_{\bullet},y_{\bullet})}{\partial y_{\bullet}^2}\right]=\mathrm{K_R}\sum_{i=1}^{\mathrm{N}}\left(\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\tag{27}$$

$$F_{x_{\bullet}y_{\bullet}}=-\mathbb{E}\left[\frac{\partial^2 l(\tilde{\boldsymbol{t}}|x_{\bullet},y_{\bullet})}{\partial y_{\bullet}\partial x_{\bullet}}\right]=\mathrm{K_R}\sum_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})(y_i-y_{\bullet})}{d_{i\leftrightarrow\bullet}^4}\right)\tag{28}$$

where we have set

$$\mathrm{K_R}=\left[\frac{100\eta^2}{\sigma_{\mathrm{R}}^2\cdot(\ln(10))^2}\right]\tag{29}$$

And finally we get the CRLB

$$\begin{aligned}
&\mathbb{E}\left[({}^{E}\tilde{x}_{\bullet}-x_{\bullet})^2+({}^{E}\tilde{y}_{\bullet}-y_{\bullet})^2\right]\\
&\geq\frac{\mathrm{K_R}^{-1}\left\{\sum\limits_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)+\sum\limits_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\right\}}{\sum\limits_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\cdot\sum\limits_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)-\left[\sum\limits_{i=1}^{\mathrm{N}}\left(\frac{(x_i-x_{\bullet})(y_i-y_{\bullet})}{d_{i\leftrightarrow\bullet}^4}\right)\right]^2}
\end{aligned}\tag{30}$$

IV. Hybrid ToA/RSS

In a last step, we derive the CRLB for the combined system, which uses a combination of $\mathrm{N_T}$ ToA and $\mathrm{N_R}$ RSS readings. The individual p.d.f. from eq. (4) and (5) as well as from eq. (23) and (24) are used to form the joint p.d.f., where again stochastic independence is assumed

$$f(\tilde{\boldsymbol{t}},\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})=\left[\prod_{i=1}^{\mathrm{N_T}}f(\tilde{t}_{i\leftrightarrow\bullet}|x_{\bullet},y_{\bullet})\right]\cdot\left[\prod_{i=1}^{\mathrm{N_R}}f(\tilde{P}_{i\leftrightarrow\bullet}|x_{\bullet},y_{\bullet})\right]\tag{31}$$

Since the logarithm of a product is equal to the sum of the individual logarithms of the factors, the logarithmic p.d.f. reads

$$l(\tilde{\boldsymbol{t}},\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet}):=\ln\Big(f(\tilde{\boldsymbol{t}},\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})\Big)=l(\tilde{\boldsymbol{t}}|x_{\bullet},y_{\bullet})+l(\tilde{\boldsymbol{P}}|x_{\bullet},y_{\bullet})\tag{32}$$

Thus, we determine the elements of the Fisher information matrix as the sum of the elements for ToA and RSS. In doing so, we finally obtain the CRLB of the hybrid system

$$\begin{aligned}
&\mathbb{E}\left[({}^{E}\tilde{x}_{\bullet}-x_{\bullet})^2+({}^{E}\tilde{y}_{\bullet}-y_{\bullet})^2\right]\\
&\geq\frac{\left\{\begin{array}{c}\mathrm{K_S}\sum\limits_{i=1}^{\mathrm{N_T}}\left[\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]+\mathrm{K_R}\sum\limits_{i=1}^{\mathrm{N_R}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)+\\ \mathrm{K_S}\sum\limits_{i=1}^{\mathrm{N_T}}\left[\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]+\mathrm{K_R}\sum\limits_{i=1}^{\mathrm{N_R}}\left(\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\end{array}\right\}}{\left\{\begin{array}{c}\left\{\mathrm{K_S}\sum\limits_{i=1}^{\mathrm{N_T}}\left[\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]+\mathrm{K_R}\sum\limits_{i=1}^{\mathrm{N_R}}\left(\frac{(x_i-x_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\right\}\cdot\\ \left\{\mathrm{K_S}\sum\limits_{i=1}^{\mathrm{N_T}}\left[\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^2}\right]+\mathrm{K_R}\sum\limits_{i=1}^{\mathrm{N_R}}\left(\frac{(y_i-y_{\bullet})^2}{d_{i\leftrightarrow\bullet}^4}\right)\right\}-\\ \left\{\mathrm{K_S}\sum\limits_{i=1}^{\mathrm{N_T}}\left[\frac{(x_i-x_{\bullet})(y_i-y_{\bullet})}{d_{i\leftrightarrow\bullet}^2}\right]+\mathrm{K_R}\sum\limits_{i=1}^{\mathrm{N_R}}\left(\frac{(x_i-x_{\bullet})(y_i-y_{\bullet})}{d_{i\leftrightarrow\bullet}^4}\right)\right\}^2\end{array}\right\}}
\end{aligned}\tag{33}$$

V. Evaluations

In this section, we compare the theoretical CRLB with the help of evaluations. As a basis, we choose a scenario of size $10\,\mathrm{m}\times10\,\mathrm{m}$. For localization, we employ four BS, which are put into the corners. Characteristic values are selected for the parameters, i.e. $(\mathrm{c}\cdot\sigma_{\mathrm{T}})=1.8\,m$ [12], $\sigma_{\mathrm{R}}^2=9$ and $\eta=2.5$ [1], [13], [14] (σ_{R}^2 and η are unitless).

The figures 1a and 1b illustrate the bounds for ToA and RSS over the $10\,\mathrm{m}\times10\,\mathrm{m}$ scenario. As presumed, more

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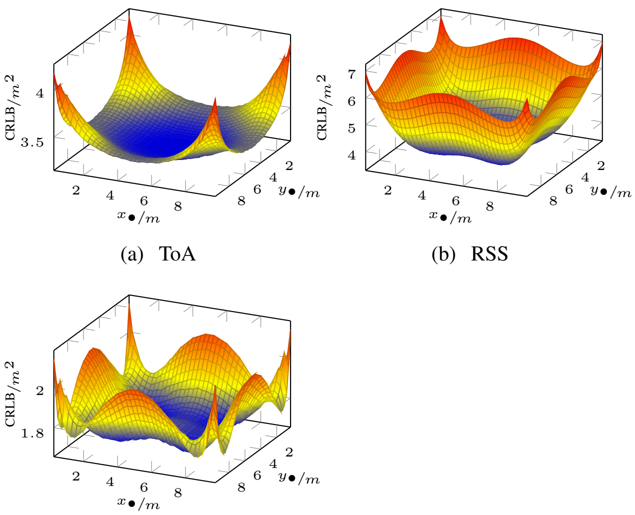
accurate results can be expected for ToA, since the CRLB for ToA is below the CLRB for RSS for all points in the scenario.

In figure 1c the underlying ToA measurements are extended with regards to incorporating the four additional readings of the RSS. As already mentioned, these are determined in most cases anyway, thus no additional hardware is necessary. The evaluations reveal, that the CRLB of ToA/RSS is approximately half of ToA, for our selected parameter values, which reflect a typical use case. Therefore, a considerable accuracy improvement is expected by incorporating these RSS readings.



(a) ToA

(b) RSS

(c) ToA/RSS

Fig. 1. CRLB for ToA, RSS and ToA/RSS

VI. Conclusion and Future Work

This paper determined the theoretical bounds for any unbiased estimator for common positioning techniques. As expected, RSS localization is inferior compared to using ToA. Our findings were supported by evaluations, where typical associated parameter values were selected. In a further step, the CRLB for the combined ToA/RSS variant was derived. Comparing all three approaches, the evaluation demonstrate that the CRLB for the hybrid technique is approximately half of the CRLB for ToA. In summary, these theoretical findings reveal that the additional application of RSS readings in a ToA system can lead to considerable performance improvements.

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